

Normal Series and Subnormal Series.

Definition A finite group system of subgroups of a group G such that

$$G = G_0 \supset G_1 \supset G_2 \supset \dots \supset G_m = \{e\}$$

is called Subnormal series of group G if every subgroup G_n is a proper normal subgroup of G_{n-1} for $n=1, 2, 3, \dots, m$ — (1)

It should be noted that G_1 is a normal subgroup of G , G_2 is a normal subgroup of G_1 , but not necessarily of G and so on.

Length The number of factors in a subnormal series is called length of the series.

Normal Series — A subnormal

series $G = G_0 \supset G_1 \supset G_2 \supset \dots \supset G_m = \{e\}$ of a group G is called normal series if G_n is normal in $G \forall n$.

Example i) If G is an abelian group then its subnormal series and normal series coincide.

Factors Let G be a group, the factor groups (Quotient groups)

$$G/H_1, H_1/H_2, \dots, H_{m-1}/\{e\}$$

are called factors of the subnormal series.

$$G = H_0 \supset H_1 \supset H_2 \supset \dots \supset H_m = \{e\}$$